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XX.—*The Bifilar Magnetometer, its Errors and Corrections, including the Determination of the Temperature Coefficient for the Bifilar employed in the Colonial Observatories.* By JOHN ALLAN BROUN, F.R.S., Director of the Trevandrum Observatory.

(Read 4th February 1861.)

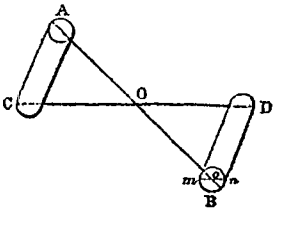
In 1845, a paper by me on the balance magnetometer was read to the Royal Society of Edinburgh (see Trans. vol. xvi. p. 67), which contained an examination of some of the difficulties to be considered and overcome in relation to that instrument. No similar examination has yet appeared, as far as I am aware, of the bifilar magnetometer. When it is considered that the value of the results obtained from this instrument in so many observatories is dependent on an exact knowledge and elimination or correction of its errors; and, as will appear hereafter, that the temperature coefficients employed in the discussion of the Colonial observations are in some cases so erroneous that the uncorrected observations would have been nearer the truth than after correction, the following communication may not appear unnecessary.

2. The bifilar magnetometer, as devised by Dr LLOYD, was employed in all the British and Colonial observatories with one exception; a description of the instrument will be found in the introductions to the "Makerstoun Observations," forming part of the Society's Transactions.

3. It is well known that if W represent the weight of the magnet and its appendages, m the magnetic moment, X the horizontal force of the earth's magnetism, l the length of the wires, a and b their intervals above and below, i the angle which they make with the vertical, v the angle of twist, and u the angle which the magnet makes with the magnetic meridian; then,

$$mX \sin u = \frac{Wab}{l \cos i} \sin v, \quad . \quad . \quad . \quad (1.)$$

This, however, is on the supposition that the wires are lines without elasticity; for if AC, BD, represent the vertical projections of the wires on the horizontal plane, then (to consider one wire only) it is evident that the upper end of the wire BD has turned through the angle v more than the lower end. Let us suppose that the wire BD is suspended alone with a weight equal $\frac{W}{2}$, whose magnetic moment $= m$ (or whose ratio to m is known); let the upper extremity of the wire be turned through the angle v, and the magnet be moved from the meridian through the angle δ, then, if the torsion force for unity of arc be p, we have



$$p(v - \delta) = mX \sin \delta,$$

or,

$$pv = mX \sin \delta, \quad . \quad . \quad . \quad . \quad . \quad (2).$$

4. If we now consider the wire BD in the bifilar arrangement, and suppose it moveable round an axis at o, so that mn which, when the wires were vertical, was parallel to OD, still remains parallel to OD, and we then turn B round the axis o, so that mn shall be in the same line as AB; the moment of this twist through the angle v, for both wires, in turning the magnet round O, must be added to equation (1), and we have, if $u = 90°$,

$$mX = \frac{Wab}{l \cos i} \sin v + 2pv \quad . \quad . \quad . \quad (3).$$

$$= G \sin v + 2mX \sin \delta', \quad . \quad . \quad . \quad (4).$$

where $G = \frac{Wab}{l \cos i}$

If we assume

$$mX = G \sin v', \quad . \quad . \quad . \quad (5).$$

neglecting the variation of i, we have

$$\sin \delta' 2mX = G \cos \frac{v + v'}{2} \sin (v - v'), \quad . \quad . \quad (6).$$

Whence, approximately,

$$\sin (v - v') = \frac{2 \sin \delta'}{\cot v} \quad . \quad . \quad . \quad (7).$$

If $\sin (v - v') = \tan \beta$, then,

$$mX = G (\sin v - \cos v \tan \beta) = \frac{G \sin (v - \beta)}{\cos \beta}, \text{ since } (v' > v) \quad (8).$$

and,

$$\frac{\Delta X}{X} = \cot (v + \beta) \Delta v \quad . \quad . \quad . \quad . \quad . \quad (9).$$

whereas by equation (1), $\frac{\Delta X}{X} = \cot v \Delta v \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$

If we know the torsion force of the wires, β may always be computed from equation (7). The following experiments have been made to determine the probable value of β.

5. *Torsion Force of Wires.*—Silver wire used in Grubb's bifilar, 0·007 inch diameter; magnet suspended whose moment = m (= 9·0 English units approximately), and weight = $\frac{W}{2}$, when it was found that 1° of torsion gave $\delta' = 1'·56$. If $v = 60°$, then by equation (7),

$$v - v' = 5° 22'$$

6. This implies an error in the unit coefficient so much greater than is shown by the different methods of determining it, that I made the following experiments for its verification. The same magnet suspended with the same wire as bifilar, $v = 47° 47'$ (weight = W), magnet at right angles to the magnetic meridian; *one* wire was turned through angles of 10° right and left; the corresponding changes of scale reading were found, for 1° of torsion = 1′·65; therefore, for two wires and for 47° 47′ of torsion, $v' - v = 2° 37'·6$ nearly. By equation (7) we find,

$$2\delta' = 2° 21'·6,$$

whereas by experiments with the magnet in the meridian, we find

$$\Delta\delta' = 2 \times 47{\cdot}8 \times 1'{\cdot}56 = 2^\circ\ 29'{\cdot}2.$$

7. In this experiment the angles of torsion with the bifilar wire were only estimated by means of a fibre attached to the wire below the point of suspension; but other experiments were made with a more perfect apparatus; the ends of the wires were passed through holes in a brass plate of such diameter as to allow them to turn freely without any lateral displacement, and the ends were soldered into two brass pins with points, which indicated the amount of rotation of each wire on small circles graduated on the brass suspension plate, having the wire holes for centres. In this way some accurate observations were made, of which the following is the summary:—

Magnet ($m = 7{\cdot}7$ nearly) suspended by one wire in the meridian δ' for 1° of torsion, $= 0'{\cdot}433$
... ... two wires at right angles Δv ... ... $= 1{\cdot}40$,

the latter being for two wires, $v = 57^\circ\ 13'$, and $v' - v = 1^\circ\ 20'$; then,

By equation (7), $\delta' = 26'{\cdot}4$;
By experiment, $\delta' = 24{\cdot}8$.

Magnet ($m = 2{\cdot}35$) suspended by one wire in the meridian, δ' for 1° of torsion, $= 1{\cdot}42$;
... ... two wires at right angles to meridian, Δv for 1° of torsion, . . . $= 0{\cdot}74$,

the latter as before being for the torsion of both wires, $v = 14^\circ\ 52'$, $v' - v = 11'$; then,

By equation (7), $\delta' = 21'{\cdot}7$;
By experiment, $\delta' = 21{\cdot}1$.

In all instances, the confirmation is as near as could be expected.

8. Equation (3), however, is on the assumption that the wires are wholly without twist before the torsion circle is turned; a supposition wholly gratuitous in fact, since the wires have generally been wound on bobbins before use, and the twist thus introduced is rarely ever wholly removed. Trials at Makerstoun in 1841, with all the conditions as equal as possible, showed that the angle v varied about 1° (one degree), according as the torsion circle was turned to the right or to the left.*

9. *Unit Coefficient.*—In 1846 I employed a new method, which was free from the objections applying to the torsion circle observations, and in which the bifilar magnet was deflected by another placed at known distances.† I have also since then employed the following processes. If we make W, a, b, or l to vary considerably in the hypothetical equation (1.), and take the differences, a given variation of $W = \pm\Delta W$, of $a = \pm\Delta a$, of $l = \pm\Delta l$, will be connected with the observed variation of $v = \Delta v$ ($u = 90^\circ$) by the equations.‡

* See Introduction to Makerstoun Observations, Trans. Roy. Soc. Ed., vol. xvii. part 1. p. 27

† Introduction to Mak. Obs., 1843, p. lxv.; 1844, p. xxviii.; 1845, p. xxvii.

‡ I have only lately learned that Mr Brooke has employed the process by weights for his self-registering instruments (Phil. Trans., p. 85, 1850).

$$\text{Cot } v \tan \Delta v = \frac{\Delta W}{W \pm \Delta W} = \frac{\Delta a}{a \pm \Delta a} = \frac{\Delta l}{l}$$

or, since Δv is always small $= n\,\alpha$, where α is the arc value of one division of the scale, and n the observed number of scale divisions of deflection, then

$$\alpha \cot v = k = \frac{\Delta W}{n\,(W \pm \Delta W)} = \frac{\Delta a}{n'\,(a \pm \Delta a)} = \frac{\Delta l}{n''\,l}$$

To these methods may be added that given by GAUSS, depending on the time of vibration of the magnet.

10. The following table contains a summary of the results for the value of k obtained for different bifilars by the processes indicated above.

TABLE I.—VALUES OF THE BIFILAR UNIT COEFFICIENT k BY DIFFERENT METHODS.

No.	Date.	Instrument.	Method.	Coefficient k.
				0·000 (prefix).
1	May 1847,	Grubb's bifilar, Silver wires,	Deflections 3 dist. 4 in. magnet,	1351
			„ „ 15 „	1356
			Comparisons with Unifilar force inst.,	1353
			Angle of torsion,	1251
2	Oct. 1852,	Grubb's bifilar, Silver wires,	Weights $\Delta W = +33{\cdot}0$ grains,	1365
			„ $\Delta W = +103{\cdot}1$ „	1367
			„ $\Delta W = +136{\cdot}1$ „	1370
			„ $\Delta W = -72{\cdot}32$ „	1368
			Intervals $\Delta a = +0{\cdot}01039$ in.,	137
			„ $\Delta a = -0{\cdot}01039$	124*
			Deflections 3 distances,	1369
3	Sept. 1854,	Grubb's bifilar, Platinum wires,	Deflections, 3·125 feet,	1503
			„ 2·625 „	1503
			„ 2·125 „	1503
			Weights $\Delta W = -72{\cdot}0$ gr. and $+100$ gr.	1505
			Angle of torsion,	138
4	Dec. 1854,	Idem,	Deflections 3 distances,	1503†
5	Sept. 1854,	Adie's bifilar, No. 3, Platinum wires,	Deflections 3 distances,	0578
			Angle of torsion,	0574
6	Nov. 1854,	Adie's bifilar, No. 1, Platinum wires,	Weights $\Delta W = \pm 20$ gr.,	0700
			Angle of torsion,	0692
7	Dec. 1854,	Adie's bifilar, No. 3, Platinum wires,	Weights $\Delta W = \pm 41{\cdot}5$ gr.	0513
			Deflections 3 distances,	0510
			Angle of torsion,	0501
8	May 1855,	Adie's bifilar, No. 1, Platinum wires,	Deflections 3 distances,	0451
			Weights $\Delta W = -34{\cdot}8$ gr. and $+30$ gr.,	0451
			Angle of torsion,	0449
			Horizontal vibrations,	0436

No. 1, Makerstoun; Nos. 2 to 7, Trevandrum Observatory; No. 8, Agustier Malley Observatory, Ghats of Travencore. No. 2, weight suspended 8491 grains; Nos. 3 and 4, weight suspended 6354 gr.; No. 5, weight 2960 gr.; Nos. 6, 7, and 8, weight about 2200 gr.

* The wire was found to shift in the groove of the suspension screw.

† The whole Instrument had been turned into different azimuths since the preceding determinations.

11. The results from the angle of torsion are always too little, though approaching the truth, for platinum wires with light weights. The result by vibrations shows the insufficiency of this method, without some better system of correction for arc. On the whole, it appears that by weights and deflections the same mean result is obtained very nearly; and both methods should be employed in the case of an instrument intended for a fixed observatory. The experiments by weights should, however, always be made first, as there is some danger in laying on and taking off the weights of an accident (the smallest shock producing a change of position in the magnet). When an instrument is closed up, and the line passing through the centre of the bifilar at right angles to its magnetic axis is marked, the deflections can always be made without disturbing the instrument.

12. *Temperature Corrections.*—Having satisfied ourselves that we have obtained the true value of the unit coefficient, it is desirable to consider what causes may influence the members of equation (3). The quantities may evidently all vary, but the variations of W and i may be neglected, since the only known causes which can make W to vary are the moon's attraction, producing a variation in all of less than 0·000002 of the horizontal force, and therefore inappreciable in our instruments; and the varying density of the atmosphere, producing for a variation of three inches of mercury no greater change than 0·00002 of the horizontal force X; the variation of i appears in equation (9) as $\tan i\, d\, i$, a quantity of the second order; m, a, b, l, and p, however, all vary with temperature. If we differentiate with respect to temperature, we shall obtain as corrections to equation (9)

$$\frac{d\,m}{m}+\frac{d\,a}{a}+\frac{d\,b}{b}-\frac{d\,l}{l}+2\sin\delta'\frac{d\,p}{p}. \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The variation of m, the magnetic moment, has been usually obtained by Mr Christie's process; $\frac{d\,a}{a}$, $\frac{d\,b}{b}$, and $\frac{d\,l}{l}$, are the expansion coefficients of the metals separating the wires above and below, and of the wires themselves. These coefficients have been usually taken as obtained from the expansion of solid bars of these metals. My own experiments show that this is not accurate; the expansion coefficient of a wire stretched by a weight is considerably greater than for a bar of the same metal; this error, however, bears a small proportion to the whole temperature coefficient: $\frac{d\,p}{p}$ is the correction due to the variation of the elasticity of the wires with temperature.

13. If, then, we allow that the temperature coefficient obtained by placing a magnet in water of different temperatures is the same, or very nearly the same, as should be obtained when placed in air of different temperatures, we have still the effects of temperature on the wires undetermined, It is also by no means

certain that the varying temperature of the copper pillars supporting the torsion circle has no effect on the position of the magnet.

14. The following experiments will prove that we cannot at present determine the correction for the suspension of the magnet by mere theoretical considerations. Considering that the elasticity of the wires diminishes with increase of temperature, and that the twist of the wires in turning the torsion circle acts *with* the torsion force, an increase of temperature should be equivalent to a diminution of the total torsion moment, and the north end of the bifilar magnet should *approach* the north. The effect of temperature on the magnet itself being to diminish its moment, is to cause the north end to move *from* the north. The temperature coefficient should therefore be diminished by this action on the wires (excepting the case where the proper twist of the wires is greater than v, and in an opposite direction).

15. *Trevandrum Observatory, July* 1858.—Having suspended a lead cylinder, carrying a mirror (the whole weight being about 5000 grains), by the silver wires of Grubb's bifilar, a fine silk fibre from the cocoon was attached to one of the wires near the lower end, and the fibre was passed over a light copper friction wheel (whose axle was taken from a dipping needle); forty grains were suspended at the other end of the fibre. The torsion circle was then turned till the angle v was about 65°, the pull of the fibre over the friction roller being at right angles to the plane, passing through the lower ends of the two wires. The movements of the weight round the vertical axis were observed by means of a telescope and a glass scale at a distance of seven feet; the scale readings increased when the movement was direct, and one scale division was equal to 0'·327. According to hypothesis, if the torsion was direct, an increase of temperature should correspond to a retrograde movement, or a diminution of scale readings, and *vice versa*. Hourly observations were made during the months of June and July 1858; but as the slightest shocks touching the pillar of the instrument changed the position of the weight, the series was broken from day to day. In the first series all the observations comparable were obtained from July 21 to August 5, 1858. The temperature was obtained from a thermometer within the box, having its bulb resting on a brass cup.

16. A change of the daily mean scale reading from day to day throughout the period is obvious (see second column of Table II.). On the assumption that this change was regular, a correction of + 2·0 Sc. div. a day from July 21 has been applied; the reduced quantities are given in the third column. The individual results are by no means satisfactory. They show generally, however, an increase of reading for an increase of temperature; and the final result is, that +1°·0 Fahr. is equivalent to + 4·0 Sc. div., thus satisfying the hypothesis.

TABLE II.—Torsion Retrograde.

Trevandrum Observatory, Bifilar Weight.						
Date.	Mean Scale Reading.		Mean Temperature.	Mean Change.		
	Observed.	Reduced for Change.		Observed Scale Reading.	Reduced Scale Reading	Temperature.
			°			°
July 21	37·59	37·59	78·46			
„ 22	18·77	20·77	77·20	−18·82	−16·82	−1·26
„ 23	13·35	17·35	77·29	− 5·42	− 3·42	+0·09
„ 24	22·23	28·23	77·71	+ 8·88	+10·88	+0·42
„ 26	31·91	41·91	78·37	+ 9·68	+13·68	+0·66
„ 27	31·32	43·31	79·18	− 0·59	+ 1·40	+0·81
„ 28	27·11	41·11	79·04	− 4·21	− 2·20	−0·14
„ 29	26·23	42·23	79·23	− 0·88	+ 1·12	+0·19
„ 30	22·71	40·71	79·15	− 3·52	− 1·52	−0·08
„ 31	21·82	41·82	78·74	− 0·89	+ 1·11	−0·41
Aug. 2	14·81	38·81	77·27	− 7·01	− 3·01	−1·47
„ 3	13·95	39·95	77·13	− 0·86	+ 1·14	−0·14
„ 4	10·88	38·88	77·09	− 3·07	− 1·07	−0·04
„ 5	6·02	36·02	77·96	− 4·86	− 2·86	+0·87

17. On August 7 a new series were commenced, the torsion being in this case *direct;* the following are the results:—

TABLE III.—Torsion Direct.

Trevandrum Observatory, Bifilar Weight.						
Date.	Daily Mean.		Mean Temperature.	Mean Change.		
	Observed Scale reading.	Reduced for change.		Scale reading.	Scale reading reduced.	Temperature.
			°			°
Aug. 7	19·59	19·59	77·86			
„ 9	15·56	16·90	78·60	−4·03	−2·69	+0·74
„ 10	14·60	16·61	79·07	−0·96	−0·29	+0·47
„ 11	14·03	16·71	78·10	−0·57	+0·10	−0·97
„ 12	16·69	20·04	77·16	+2·66	+3·33	−0·94
„ 13	11·97	15·77	78·02	−4·92	−4·25	+0·86
„ 14	10·75	15·44	78·57	−1·02	−0·25	+0·55
„ 16	8·30	14·33	79·02	−2·45	−1·11	+0·45
„ 17	9·31	16·01	78·69	+1·01	+1·68	−0·33
„ 18	10·97	18·34	78·03	+1·66	+2·33	−0·66
„ 19	11·36	19·40	77·57	+0·39	+1·06	−0·46
„ 20	10·89	19·60	77·87	−0·47	+0·20	+0·30

The mean scale reading having changed about 9 sc. div. from the 7th to the 20th, a correction has been applied after the 7th, for the change supposed regular at the rate of 0·67 sc. div. a-day; the corrected quantities are given in the third column. A correction has been allowed for a shock to the instrument on the 11th, depending on the change at the hour of the shock; and the change 10th to 11th is on this account probably inaccurate. This series is, on the whole, consistent, an increase of temperature being accompanied by a diminution of scale reading (or *vice versa*) on every day with one exception; the final result is +1·0° Fahr., equivalent to −2·5 sc. div. This also satisfies the hypothesis.

18. Perhaps the most curious fact connected with this series of observations is, that though the *daily* mean scale reading increased, torsion retrograde, with an increase of temperature, and torsion direct, with a diminution of temperature, there was in both cases a strongly and regularly marked *hourly* variation, whose direction was not changed by the direction of the torsion, and whose amount, as related to the temperature, was much greater than that obtained from the daily means. I shall have to consider this fact at another time: it does not, however, depend upon the bifilar suspension. Whatever conclusion we may arrive at with reference to the relation of these facts to the direction of torsion, it appears to me they are sufficient to prove that any trustworthy temperature coefficient must include their causes.

19. There still remains one considerable objection to the usual method of obtaining the temperature coefficient of the bifilar magnetometer; namely, that before we can correct the observations we must be certain that we have obtained the true value of the scale unit coefficient. Should that be at all inaccurate (as it frequently has been when derived from the angle of torsion only), the corrections, even by a true temperature coefficient, will be inaccurate. Thus, if n_0 be the scale reading from the zero, k the unit, and q the temperature, coefficients, the reduced result is

$$\frac{\Delta X}{X} = n_0 k + t_0 q. \qquad (16)$$

where t_0 is the number of degrees Fahr. from the zero. If n_1 be the part of n_0 due to change of the earth's magnetism, and n_2 the part due to temperature,

$$\frac{\Delta X}{X} = n_1 k + n_2 k + t_0 q. \qquad (17)$$

the last two quantities should destroy each other if k and q be the true coefficients. If, however, $k_0 = k + \alpha$ be the true unit coefficient, then

$$n_2 k_0 = -t_0 q,$$

and

$$n_2 k = -t_0 q - \alpha \frac{q}{k_0} t_0. \qquad (18)$$

Substituting this value in equation (17),

$$\frac{\Delta X}{X} = n_1 k - \alpha \frac{q}{k_0} t \qquad (19)$$

the corrected result, which *still* varies with temperature. Where the scale unit coefficient can be accurately determined, this consideration has no value; but where we have to deal with observations already made, it is one of much importance.

20. *Temperature Coefficient.*—Several facts, such as those already indicated, relatively to the effects of temperature on the parts of the balance magnetometer, induced me, in 1842, soon after I received charge of Sir THOMAS BRISBANE'S Observatory at Makerstoun, to seek some other method of obtaining the temperature coefficient in the terms of the *scale divisions*. This I sought first by artificially heating and cooling the room in which the instruments were placed; but the effects of currents of air thus engendered were too considerable to render the method satisfactory. I then sought to determine it by comparisons of the readings at the same hours of different days, when the temperatures were different; I finally employed successfully the daily means, at first from incomplete diurnal series of observations in 1842-43. This method, by the formula

$$\frac{\Delta X}{k X} = n_1 + n_2 + \frac{q}{k} t_0,$$

gave the result accurately corrected in scale divisions, from which the *laws* of variation could be determined even when ignorant of the value of k.*

21. When this method was first used by me, I selected periods free from large disturbances, and combined the daily means in different periods.† In determining the coefficient for the colonial and other observatories, I have thought it preferable (to avoid any doubt that might arise from such selection as to the character of the final results) to obtain the coefficients from the comparisons of all the daily means *without exception*, from the commencement to the end of the periods for which observations have been published. In general, each daily mean of scale divisions and of temperature has been compared with the succeeding daily mean only; but where the variation of mean temperature from day to day has been very small, comparisons with the second, and sometimes with the third day following, have been taken. The following are the discussions for the different bifilars:—

* I first proposed to Professor JAMES D. FORBES (in a letter dated 21st November 1842) to determine the temperature coefficient from the usual hourly observations. (See Trans. Roy. Soc. Ed., Vol. XVI. p. 74.)

P.S.—Since this paper was written, I have received the third volume of the Toronto Observations (London 1857), where I am glad to perceive that General SABINE has followed the principle described in the text, for the determination of the temperature coefficient. The *mode* he has adopted is however far from accurate, including as it does irregular annual secular and instrumental changes, with too few results to render *plus* and *minus* errors equally probable; as a consequence, the coefficient obtained by him is still erroneous (1·63 sc. div., whereas I find 1·36 sc. div.). General SABINE notices my result for the Makerstoun bifilar (Adjustments, &c., part v., Toronto Observations, vol. iii.), but he omits to remark that I had pointed out, twelve years before, the necessity of applying the process to all bifilars, and that I had applied it to the very magnet (the Toronto one) under his discussion. See Trans. Roy. Soc. Ed., Vol. XVI. p. 77 (20), p. 102 (11), and curves in No. 5, Plate IV.

† See the Introduction to the Makerstoun Observations for 1844 and 1845, where the process is described.

22. Cape of Good Hope, 1841–46.—The series of published observations closed June 1846.* The values of q' obtained by myself from the daily mean uncorrected scale readings and mean temperatures were as follow :—

Magnet R. $k = 0{\cdot}000180$

Table IV.—Sums of Successive Differences of Daily Mean Bifilar Temperature ($\Sigma \Delta t$) and Scale Reading ($\Sigma \Delta$ R).

Month.		$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'.	Month.		$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'.
		°	Sc. div.				°	Sc. div.	
1841.	Dec.	33·23	52·38	1·58	1842.	Dec.	29·65	45·09	1·52
1842.	Jan.	29·27	34·31	1·17	1843.	Jan.	32·55	44·80	1·38
	Feb.	34·43	24·73	0·72		Feb.	39·95	48·19	1·21
	March.	23·14	24·43	1·04		March.	26·23	18·43	0·70
	April.	33·31	46·16	1·39		April.	Magnet	removed.	
	May.	18·81	19·10	1·02		May.	18·60	33·57	1·80
	June.	27·56	36·88	1·34		June.	35·41	43·38	1·23
	July.	25·41	44·48	1·75		July.	29·38	30·56	1·04
	Aug.	20·13	13·69	0·68		Aug.	43·60	64·55	1·48
	Sept.	16·99	9·52	0·56		Sept.	28·30	25·14	0·89
	Oct.	39·70	50·65	1·28		Oct.,	33·01	25·86	0·78
	Nov.	26·48	27·44	1·04		Nov.	Readjusted.		

The resulting value is

$$q' = \frac{\Sigma \Delta \mathrm{R}}{\Sigma \Delta t} = \frac{763{\cdot}34}{645{\cdot}14} = 1{\cdot}183 \text{ sc. div.; mean temperature, } 63°{\cdot}6.$$

As the magnet was removed for temperature experiments in April 1843, the value of q' after the readjustment may be determined apart. We find for May to October 1843,

$$q' = \frac{223{\cdot}06}{188{\cdot}30} = 1{\cdot}185 \text{ scale divisions.}$$

23. It should be repeated here, that the extent of agreement or disagreement in the partial results for q' given in the above table, is not to be considered as deciding the accuracy of this method, since the variation of these partial results is chiefly due to days of great magnetic disturbance, and no day has been rejected, *however great its irregularity*, in these calculations. These partial results give the probable error of the final value of q' to be, 0·05 sc. div.; or $q' = 1{\cdot}183$ scale divisions, with a probable error of 0·05 sc. div.

* Magnetic Observations, Cape of Good Hope, vol. i., printed under the superintendence of Lieut.-Colonel E. Sabine.

Whence, since

$$k = 0{\cdot}000180,$$
$$q'k = [q] = 0{\cdot}000213.$$

The value of q determined by hot water experiments was found for the magnet R,

$$q = 0{\cdot}000258 \text{ at the temperature of } 48^\circ$$
$$q = 0{\cdot}000304 \quad \text{,,} \quad 77^\circ$$

the value of q varying apparently with the temperature, as occurs with some bars. The value of q at the mean temperature of the bar for the months in Table IV. is

$$q = 0{\cdot}000290 \text{ at the mean temperature of } 63^\circ{\cdot}6;$$

whence $$[q] = 0{\cdot}734\ q.$$

The same value of q' ($=1{\cdot}18$) has been employed by me (for the discussions given in another paper) in the corrections for all temperatures.

24. *November* 1843 *to September* 1844.—New adjustment of magnet R,

$$k = 0{\cdot}00035.$$

TABLE V.—SUMS OF SUCCESSIVE DIFFERENCES OF DAILY MEANS.

Month.		$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'.	Month.		$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'.
			Sc. div.	Sc. div.				Sc. div.	Sc. div.
1843.	Nov.	27°74	11·83	0·43	1844.	May.	24°87	17·01	0·68
	Dec.	32·24	21·01	0·65		June.	25·55	12·11	0·47
1844.	Jan.	33·75	17·12	0·51		July.	24·41	14·15	0·58
	Feb.	33·54	27·07	0·81		Aug.	22·71	15·07	0·66
	March.	28·84	18·42	0·64		Sept.	25·74	13·54	0·52
	April.	31·12	14·89	0·48		Oct.	New	Magnet	C_2

The resulting value, the mean temperature of the bar being 67°, is

$$q' = \frac{\Sigma \Delta R}{\Sigma \Delta t} = \frac{182{\cdot}22}{310{\cdot}51} = 0{\cdot}587 \text{ sc. div.},$$

with a probable error of 0·03 sc. div. Whence,

$$[q] = 0{\cdot}000205;$$

and since,

$$q = 0{\cdot}000287 \text{ at the temperature of } 67^\circ$$
$$[q] = 0{\cdot}714\ q.$$

As it is quite possible that the temperature coefficient for the same bar might be different for $k = 0{\cdot}00018$ and $k = 0{\cdot}00035$, the coefficient $q' = 0{\cdot}587$ sc. div., has been employed by me from November 1843 to September 1844. It is rendered probable by the near agreement of the values of $[q]$ in these two adjustments, that the difference from the value of q is not due to error of the coefficient k.

25. *November* 1844 *to June* 1846.—Magnet C_2, new adjustment,

$$k = 0{\cdot}00022.$$

TABLE VI.—Sums of Differences of Bifilar Daily Means.

Month.		$\Sigma\Delta t$.	$\Sigma\Delta R$.	q'.	Month.		$\Sigma\Delta t$	$\Sigma\Delta R$.	q'.
			Sc. div.	Sc. div.			°	Sc. div.	Sc. div.
1844.	Nov.	34·46	13·79	0·40	1845.	Sept.	19·41	5·12	0·26
	Dec.	38·03	21·01	0·55		Oct.	32·39	7·55	0·23
1845.	Jan.	35·44	6·50	0·18		Nov.	33·36	13·70	0·41
	Feb.	23·90	11·59	0·49		Dec.	32·59	38·77	1·19
	March.	31·55	12·51	0·40	1846.	Jan.	37·03	24·47	0·66
	April.	28·03	2·92	0·10		Feb.	24·78	15·90	0·64
	May.	27·04	14·18	0·52		March.	33·04	11·63	0·35
	June.	26·73	2·66	0·10		April.	32·09	4·46	0·14
	July.	27·71	2·44	0·09		May.	23·47	16·41	0·70
	Aug.	27·87	25·06	0·90		June.	17·32	6·08	0·35

The final value is

$$q' = \frac{256{\cdot}75}{586{\cdot}24} = 0{\cdot}438 \text{ sc. div., with a probable error of } 0{\cdot}04 \text{ sc. div.};$$

whence, since

$$k = 0{\cdot}00022$$
$$[q] = 0{\cdot}000096$$

By water experiments,

$$q = 0{\cdot}000220$$

Therefore,

$$[q] = 0{\cdot}484\ q.$$

26. St Helena, 1842–43.—The series of published observations in my possession extends till December 1843.* As the daily differences of temperature are small, the sums for each two months of the same name in 1842–43 are given; and as the majority of the daily differences of temperature were under half a degree Fahr., the sums were also taken for those days only for which the mean temperature of the following day differed half a degree or more. The results by both methods are given in the following Table:—

* Printed under the superintendence of Lieut.-Colonel E. Sabine.

TABLE VII.—Sums of Differences of Bifilar Daily Means at St Helena for each Month, 1842–43.

Months, 1842–43.	Mean Temperature of Bifilar.	All.			Above 0°·5 Fahr.		
		$\Sigma\Delta t$.	$\Sigma\Delta R$.	q'	$\Sigma\Delta t$.	$\Sigma\Delta R$.	q'
	°	°	Sc. div.	Sc. div.	°	Sc. div.	Sc. div.
January, .	67·6	22·50	46·37	2·06	15·94	22·54	1·41
February, .	69·4	21·92	33·65	1·54	16·68	19·38	1·17
March, . .	69·8	26·49	47·11	1·78	21·01	32·38	1·54
April, . .	68·7	20·91	1·22	0·06	14·07	8·32	0·59
May, . .	66·1	23·82	17·63	0·74	16·18	13·98	0·86
June, . .	62·5	25·45	21·04	0·83	18·64	14·09	0·76
July, . .	60·8	25·08	−14·31	−0·57	18·88	6·80	0·36
August, . .	60·6	24·63	13·79	0·56	17·83	15·63	0·88
September, .	60·7	24·89	26·72	1·07	17·89	18·81	1·05
October, .	61·3	21·36	38·67	1·81	12·83	16·46	1·29
November, .	63·3	23·01	29·17	1·27	14·83	15·65	1·06
December, .	65·2	27·57	33·02	1·20	23·02	26·06	1·13

The resulting values are:—

From all, 1842, $q' = \frac{142{\cdot}35}{145{\cdot}25} = 0{\cdot}980$ scale divisions.

... 1843, $q' = \frac{151{\cdot}73}{142{\cdot}38} = 1{\cdot}066$ ... ...

Both years, $q' = \frac{294{\cdot}08}{287{\cdot}63} = 1{\cdot}02$ ... ...

From all above 0·°5 Fahr. 1842, $q' = \frac{107{\cdot}62}{102{\cdot}91} = 1{\cdot}046$ ... ...

... ... 1843, $q' = \frac{102{\cdot}85}{104{\cdot}89} = 0{\cdot}980$ ... ...

Both years, $q' = \frac{210{\cdot}47}{207{\cdot}80} = 1{\cdot}013$ sc. div.; probable error = 0·071.

27. An examination of the values of q' for the different months seems to show a variation with the season; the variation is shown also in the results for the separate years. If we discuss the quantities in the last column of Table VII. by the method of least squares with reference to the mean temperatures of the bifilar, we find the probable variation of q' for 1° Fahr. to be 0·0371 sc. div.; or if t be the temperature of the bifilar, that we may put,

$$q' = 0{\cdot}84 + (t - 60)\,0{\cdot}0371.$$

Therefore at the mean temperature of bifilar 65°

$q' = 1{\cdot}02$ sc. div., with a probable error of 0·06 sc. div.

The coefficient for the temperature variation of q' is very large, probably three times greater than has ever been found by water experiments for any magnet:

while it is pointed out that the series is not sufficiently good to give this result for the variation of q' a great value, it should not be forgotten that the rocks forming St Helena are of a highly magnetic nature, and that their magnetic force varies with the temperature.*

28. The coefficient $q' = 1{\cdot}02$ has been employed by me throughout. When $k = 0{\cdot}000189$, we find

$$[q] = 0{\cdot}000193.$$

The value of q was found by water experiments between the temperatures of 60° to 90° Fahr. to be

$$q = 0{\cdot}000276.$$

I am not certain that this includes the coefficient for the expansion of the wires, but it is suggested† that, on account of the variation of q with temperature, the true value might be more nearly

$$q = 0{\cdot}000260.$$

The value employed in the corrections of the fortnightly means in the "St Helena Observations"‡ seems to have varied; in 1842 and 1843 it was 1·47 sc. div. (=0·000278); in 1844 it was about 1·12 sc. div. =0·000213; and in 1845 about 1·29 sc. div. = 0·000245. Assuming the value by water experiments = 0·000260, we have

$$[q] = 0{\cdot}742\ q.$$

29. HOBARTON, 1842–48.—The continuous observations (without weekly adjustments) commenced April 1842, and the observations in my possession terminate with September 1848.§ The torsion circle was turned 1st April 1846, causing a change of the coefficient k, and therefore of q'. The following are the results:—

174. April 1842 to June 1843. $k = 0{\cdot}000120$.

TABLE VIII.—SUMS OF DIFFERENCES OF BIFILAR DAILY MEANS AT HOBARTON.

Month.	$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'	Month.	$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'
	°	Sc. div.	Sc. div.		°	Sc. div.	Sc. div.
1842, April	59·95	94·42	1·57	1842, Dec.	58·33	103·14	1·77
May	61·68	102·43	1·66	1843, Jan.	60·46	97·44	1·61
June	61·78	81·76	1·32	Feb.	65·69	95·58	1·45
July	57·10	113·48	1·99	Mar.	65·59	108·00	1·65
Aug.	50·09	56·02	1·12	April	72·44	111·49	1·54
Sept.	40·00	69·62	1·74	May	45·84	58·90	1·29
Oct.	50·45	64·28	1·27	June	17·65	18·62	1·05
Nov.	71·96	100·56	1·39	Readjustment.			

* At least I have found this to be the case with magnetic rocks in South India.

† Adjustments, &c., "St Helena Observations," vol. i., p. 34.

‡ *Ibid.* p. 35. This Table has some inaccuracies. The coefficient varies, and the temperature zero is not always 60°, as stated.

§ Hobarton Observations, vols. i., ii., and iii., printed under the superintendence of Lieut.-Colonel E. SABINE.

The final result is,

$$q' = \frac{\Sigma \Delta R}{\Sigma \Delta t} = \frac{1275{\cdot}74}{839{\cdot}01} = 1{\cdot}520 \text{ sc. div.; with a probable error of } 0{\cdot}049 \text{ sc. div.}$$

$$[q] = 0{\cdot}000182$$

By water experiments,

$$q = 0{\cdot}000234$$

Whence

$$[q] = 0{\cdot}778\ q.$$

30. In order to render the following determinations comparable, the sums of scale readings after 1st April 1846 have been multiplied by 0·948 to reduce them to the value of k before that date.

August 1843 to March 1846, $k = 0{\cdot}000229$
April 1846 to September 1848, $k = 0{\cdot}000217$

TABLE IX.—Sums of Differences of Bifilar Daily Means at Hobarton, August 1843 to September 1848.

Month.	$\Sigma \Delta t$.	$\Sigma \Delta$R.	q'	Month.	$\Delta \Sigma t$.	$\Sigma \Delta$ R.	q'	Month.	$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'
1843				1845	°	Sc. div.	Sc. div.	1847	°	Sc. div.	Sc. div.
Jan.	...	...	...	Jan.	54·66	26·93	0·49	Jan.	38·58	18·90	0·49
Feb.	...	...	...	Feb.	35·16	29·51	0·81	Feb.	38·94	18·36	0·47
Mar.	...	...	...	Mar.	40·35	26·02	0·64	Mar.	48·30	40·00	0·82
April	...	...	...	April	40·71	44·43	1·09	April	35·10	29·23	0·83
May	...	...	...	May	37·21	21·66	0·58	May	35·64	17·52	0·50
June	...	...	...	June	34·30	29·96	0·87	June	23·20	16·57	0·71
July	...	...	...	July	35·50	32·66	0·92	July	26·51	21·88	0·82
Aug.	42·94	30·80	0·71	Aug.	27·26	24·09	0·88	Aug.	33·29	21·98	0·66
Sept.	39·21	31·20	0·80	Sept.	42·44	21·61	0·51	Sept.	40·00	34·04	0·85
Oct.	58·25	37·33	0·64	Oct.	55·73	43·35	0·78	Oct.	39·79	34·48	0·86
Nov.	42·72	32·99	0·77	Nov.	54·86	45·47	0·83	Nov.	38·87	18·95	0·48
Dec.	38·21	25·44	0·67	Dec.	39·99	37·15	0·93	Dec.	50·77	−0·74	−0·02
1844				1846				1848			
Jan.	49·27	30·22	0·61	Jan.	52·77	41·39	0·78	Jan.	54·93	38·91	0·71
Feb.	48·08	31·68	0·66	Feb.	51·57	48·70	0·94	Feb.	50·99	32·11	0·63
Mar.	53·21	45·11	0·85	Mar.	52·54	66·25	1·26	Mar.	40·52	32·27	0·80
April	40·05	27·43	0·68	April	39·98	32·45	0·81	April	58·25	23·35	0·40
May	38·65	29·17	0·75	May	31·96	12·48	0·39	May	27·38	19·15	0·70
June	27·46	21·61	0·79	June	30·82	22·36	0·73	June	23·83	12·25	0·51
July	29·56	22·05	0·75	July	20·94	7·87	0·38	July	22·69	5·56	0·25
Aug.	21·13	11·38	0·54	Aug.	34·41	28·01	0·82	Aug.	21·12	27·94	1·33
Sept.	25·53	20·44	0·80	Sept.	42·58	30·40	0·71	Sept.	36·69	16·09	0·44
Oct.	37·23	28·71	0·77	Oct.	51·58	26·77	0·52	...	...	...	...
Nov.	40·97	18·64	0·45	Nov.	42·58	31·09	0·73	...	...	...	...
Dec.	46·03	22·35	0·48	Dec.	48·48	27·15	0·56	...	...	...	...

The final result is—

For $k = 0{\cdot}000229$

$$q' = \frac{\Sigma \Delta R}{\Sigma \Delta t} = \frac{1703{\cdot}11}{2462{\cdot}27} = 0{\cdot}692 \text{ sc. div.; probable error, } 0{\cdot}017.$$

For $k = 0{\cdot}000217$

$$q' = 0{\cdot}730 \text{ sc. div.}$$

$$[q] = 0{\cdot}000158.$$

52575

By water experiments,

$$q = 0{\cdot}000230$$

Therefore,

$$[q] = 0{\cdot}689\ q.$$

31. The quantities in Table IX. have been combined into periods of twelve months, and the following results obtained:—

1843, August—1844, July.	$q' = 0{\cdot}72$	sc. div.,	probable error	$= 0{\cdot}02$	sc. div.
1844, ... 1845, ...	$= 0{\cdot}80$	...	...	...	$= 0{\cdot}04$	...
1845, ... 1846, ...	$= 0{\cdot}66$	...	...	...	$= 0{\cdot}03$	...
1847, ... 1848, Sept.	$= 0{\cdot}59$	...	...	...	$= 0{\cdot}06$	...

32. The sums having been arranged according to the months, the following are the results, each being a mean of *five*, excepting those for August and September, which are the mean of six months:—

TABLE X.—Sums of Differences of Bifilar Means for each Month.

Month.	Mean Temperature of Bifilar.	q'	Probable Error.
	°	Sc. div.	Sc. div.
January,	63·90	0·62	0·06
February,	62·92	0·71	0·06
March,	61·63	0·89	0·07
April,	57·60	0·73	0·08
May,	53·19	0·59	0·05
June,	49·36	0·74	0·04
July,	47·73	0·67	0·12
August,	49·31	0·80	0·08
September,	52·44	0·68	0·06
October,	55·77	0·70	0·05
November,	59·04	0·67	0·07
December,	62·96	0·50	0·11

These values have been discussed by least squares, to determine if q' varies with t the temperature; and the coefficient for $+1°$ Fahr. was found $= -0{\cdot}004$ sc. div., or nearly zero; but the partial values of q' are too irregular to give the conclusion any great weight. The irregularities in the value of q' can be traced in general to large disturbances.

33. Trevandrum, 1842–45.*—The observations discussed include the years 1842–45 only. The results are as follows:—

1842, Magnet No. 3, $k = 0{\cdot}000135$.

* Observations in manuscript.

TABLE XI.—Sums of Differences of Bifilar Daily Means at Trevandrum, 1842.

Month.	$\Sigma\Delta t$	$\Sigma\Delta R$	q'	Month.	$\Sigma\Delta t$	$\Sigma\Delta R$	q'
	°	Sc. div.	Sc. div.		°	Sc. div.	Sc. div.
January, . .	16·83	42·0	2·50	July, . . .	21·67	50·2	2·32
February, . .	15·43	35·3	2·29	August, . .	18·31	32·9	1·80
March, . .	18·54	13·2	0·71	September, .	15·29	22·0	1·44
April, . . .	17·51	34·1	1·95	October, . .	22·89	48·0	2·10
May, . . .	29·62	35·8	1·21	November, .	16·62	12·7	0·76
June, . . .	22·64	17·6	0·78	December, .	20·59	27·0	1·31

The partial results are very irregular, on account of the smallness of the differences of temperature. The final result is,—

$q' = 1{\cdot}57$ sc. div., with a probable error of 0·15 sc. div.

When the differences of the daily mean readings, which were corrected for temperature and exceeded five scale divisions, were rejected, as well as all the differences corresponding to differences of temperature under 0°·5 Fahr., the result was found,—

$$q' = 1{\cdot}56 \text{ sc. div.};$$

whence

$$[q] = 0{\cdot}000212.$$

By water experiments,

$$q = 0{\cdot}000253.$$

Therefore,

$$[q'] = 0{\cdot}838\ q.$$

34. Bar, No. 5.

January 2, 1843, to April 25, 1843, $k = 0{\cdot}000149$.
May 2, 1843, to March 30, 1844, $k = 0{\cdot}000141$.
April 1, 1844, to „ 1846, $k = 0{\cdot}000144$.

In the following table, the sums of readings have all been reduced to the value of $k = 0{\cdot}000144$.

TABLE XII.—Sums of Differences of Bifilar Daily Means at Trevandrum, 1843–45.

Months.	1843.			1844.			1845.		
	$\Sigma\Delta t$	$\Sigma\Delta R$	q'	$\Sigma\Delta t$	$\Sigma\Delta R$	q'	$\Sigma\Delta t$	$\Sigma\Delta R$	q'
	°	Sc. div.	Sc. div.	°	Sc. div.	Sc. div.	°	Sc. div.	Sc. div.
January, . . .	18·29	26·3	1·44	16·64	37·4	2·25	11·96	33·0	2·76
February, . . .	9·70	19·8	2·04	15·46	31·6	2·04	11·32	31·9	2·82
March, ...	16·26	32·3	1·99	12·31	27·3	2·22	26·49	50·8	1·92
April, ...	24·65	51·9	2·11	18·74	36·4	1·94	16·99	42·1	2·48
May, ...	34·02	79·6	2·34	21·13	50·2	2·38	21·01	50·9	2·42
June, ...	19·71	35·0	1·78	8·21	20·1	2·45	28·01	33·2	1·18
July, ...	18·66	43·6	2·34	23·43	42·7	1·82	19·28	35·0	1·82
August, ...	15·95	53·6	3·36	18·82	37·8	2·01	16·81	26·5	1·58
September, ...	10·74	26·0	2·42	14·81	16·5	1·11	11·70	8·0	0·68
October, ...	20·47	61·4	3·00	17·40	69·6	4·00	14·46	49·6	3·43
November, ...	19·07	51·2	2·68	15·47	1·9	0·12	16·21	54·1	3·34
December, ...	27·42	48·2	1·76	16·98	43·1	2·54	20·84	51·4	2·47

The final results are,

1843. $q' = \frac{\Sigma \Delta R}{\Sigma \Delta t} = \frac{528{\cdot}90}{234{\cdot}94} = 2{\cdot}25$ sc. div.; probable error = 0·11 sc. div.

1844. $q' = \frac{\Sigma \Delta R}{\Sigma \Delta t} = \frac{414{\cdot}60}{199{\cdot}40} = 2{\cdot}08$ „ „ = 0·15 „

1845. $q' = \frac{\Sigma \Delta R}{\Sigma \Delta t} = \frac{466{\cdot}50}{215{\cdot}16} = 2{\cdot}17$ „ „ = 0·18 „

All,

$$q' = \frac{\Sigma \Delta R}{\Sigma \Delta t} = \frac{1410{\cdot}00}{649{\cdot}50} = 2{\cdot}17 \text{ „ „ . = } 0{\cdot}09 \text{ „}$$

whence,

$$[q] = 0{\cdot}000312$$

In 1843 I found by water experiments,

$$q = 0{\cdot}000347.$$

Therefore,

$$[q] = 0{\cdot}899\, q.$$

35. SINGAPORE, 1842–45.* The changes of daily mean temperature by the bifilar thermometer were so small, that differences for the 1st, 2d, and 3d succeeding days were taken for the temperature coefficient. The following table contains the sums of all the differences taken three times for each of the years 1842, 1843, 1844, and 1845.

198. $k = 0{\cdot}0001946$.

TABLE XIII.—SUMS OF DIFFERENCES OF BIFILAR DAILY MEANS. SINGAPORE, 1842–45.

Month.	$\Sigma \Delta t$	$\Sigma \Delta R$	q'	Month.	$\Sigma \Delta t$	$\Sigma \Delta R$	q'
	°	Sc. div.	Sc. div.		°	Sc. div.	Sc. div.
January, . .	201·5	119·2	0·59	July, . . .	216·5	214.2	0·99
February, .	159·5	68·1	0·43	August, . .	235·4	161·2	0·68
March, . .	164·6	128·4	0·78	September, .	186·9	136·4	0·73
April, . . .	142·6	61·6	0·43	October, . .	199·5	210·6	1·05
May, . . .	220·9	89·2	0·40	November, .	168·8	111·3	0·66
June, . . .	222·4	157·9	0·71	December, .	194·7	166·7	0·86

The values of $q' = \frac{\Sigma \Delta R}{\Sigma \Delta t}$ for the different years are:—

1842. $q' = \frac{394{\cdot}5}{628{\cdot}2} = 0{\cdot}63$ sc. div.; probable error = 0·17 sc. div.

1843. $q' = \frac{358{\cdot}0}{557{\cdot}4} = 0{\cdot}64$ „ „ = 0·14 „

1844. $q' = \frac{477{\cdot}3}{537{\cdot}2} = 0{\cdot}89$ „ „ = 0·12 „

* Observations in manuscript and printed, but not published.

1845. $q' = \frac{395{\cdot}0}{590{\cdot}5} = 0{\cdot}67$ sc. div.; probable error = 0·09 sc. div.

The value resulting from Table XIII. is,

$$q' = \frac{1624{\cdot}8}{2313{\cdot}3} = 0{\cdot}702 \text{ sc. div.; probable error} = 0{\cdot}04 \text{ sc. div.;}$$

whence,

$$[q] = 0{\cdot}0001366.$$

By hot-water experiments it was found

1841, June, $q = 0{\cdot}0002686$
1841, October $q = 0{\cdot}0001667$

By applying hot sand under the suspended magnet,

$$q = 0{\cdot}0002072$$

The value adopted was,

$$q = 0{\cdot}000214$$

Therefore,

$$[q] = 0{\cdot}638\ q.$$

36. Toronto, 1842–45. The observations in 1843 till November have not been used in this determination, on account of a derangement of the bifilar wires.* The following are the results obtained :—

1842 to 1843, February 10. $k = 0{\cdot}0000743$.

Table XIV.—Sums of Differences of Bifilar Means. Toronto, 1842.

Month.	$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'	Month.	$\Sigma \Delta t$.	$\Sigma \Delta$ R.	q'
	°	Sc. div.	Sc. div.		°	Sc. div.	Sc. div.
January, .	76·55	165·73	2·16	July, . . .	44·71	161·54	3·61
February, .	72·72	189·88	2·61	August, . .	30·08	80·68	2·68
March, . .	62·77	171·53	2·73	September, .	46·81	117·75	2·51
April, . .	71·46	170·00	2·38	October, . .	35·95	77·37	2·15
May, . . .	48·46	140·02	2·89	November, .	43·99	104·77	2·38
June, . .	37·83	68·98	1·82	December, .	64·35	161·55	2·51

The final result is,

$$q' = \frac{1609{\cdot}80}{635{\cdot}68} = 2{\cdot}53 \text{ sc. div.;}\dagger \text{ probable error} = 0{\cdot}08 \text{ sc. div.;}$$

whence

$$[q] = 0{\cdot}000188.$$

* Toronto Observations, 1842–43, vols. i. and ii., printed under the superintendence of Lieutenant-Colonel E. Sabine.

† The value employed by me in 1845, to correct the Toronto bifilar observations for the annual law, was obtained from a few groups of days (the sums of differences of temperature being 117°), and was $q' = 2{\cdot}55$ sc. div., almost identical with that here obtained from the whole year.

By water experiments in

1843, $q = 0{\cdot}000260$
1849, $q = 0{\cdot}000234$

The latter value should be preferred, having been made with more care, though it is a question whether the temperature coefficient is constant. We have then, adopting the value of $q = 0{\cdot}000234$,

$$[q] = 0{\cdot}804\ q.$$

37. November 1843 to December 1845, $k = 0{\cdot}000087$.

TABLE XV.—SUMS OF DIFFERENCES OF BIFILAR MEANS. TORONTO, 1844–45.

Month.	$\Sigma\Delta t$.	$\Sigma\Delta$ R.	q'	Month.	$\Sigma\Delta t$.	$\Sigma\Delta$ R.	q'
	°	Sc. div.	Sc. div.		°	Sc. div.	Sc. div.
1843. Nov.	57·12	83·03	1·45	1844. Dec.	49·01	54·35	1·11
Dec.	47·59	64·48	1·35	1845. Jan.	71·29	98·70	1·38
1844. Jan.	83·19	86·37	1·04	Feb.	63·68	63·40	1·00
Feb.	54·19	78·87	1·46	March	48·18	53·38	1·11
March	60·59	20·28	0·33	April	54·34	84·04	1·55
April	66·64	67·50	1·01	May	82·21	117·57	1·43
May	59·30	97·93	1·65	June	56·78	68·90	1·20
June	43·27	64·93	1·50	July	59·37	61·05	1·03
July	41·38	47·25	1·14	Aug.	41·74	67·68	1·62
Aug.	40·38	57·70	1·42	Sept.	43·84	81·41	1·86
Sept.	39·23	75·82	1·93	Oct.	58·94	84·00	1·43
Oct.	58·73	85·84	1·46	Nov.	48·52	58·16	1·20
Nov.	51·14	136·78	2·67	Dec.	67·15	69·61	1·04

The final results are,

1844. $q = \frac{873{\cdot}62}{647{\cdot}05} = 1{\cdot}350$ sc. div.; probable error $= 0{\cdot}10$ sc. div.

1845. $q' = \frac{907{\cdot}90}{696{\cdot}04} = 1{\cdot}304$ „ „ $= 0{\cdot}06$ „

26 months. $q' = \frac{1929{\cdot}03}{1447{\cdot}80} = 1{\cdot}33$ „ „ $= 0{\cdot}05$ „

Therefore,

$$[q] = 0{\cdot}000116$$

By water experiments, last determination,

$$q = 0{\cdot}000234$$

and

$$[q] = 0{\cdot}496\ q.$$

The low value of q' obtained in this case, compared with that by water experiments, resembles the value obtained for the magnet C_2 at the Cape (§ 25), and both, it seems to me, indicate some derangement of the wires.

38. *P.S.*—I have discussed the observations made in 1846, 1847, and till June 30, 1848 (Toronto Obs., vol. iii., received since this paper was written), for the temperature coefficient: the observations for 1846 gave $q' = 1{\cdot}355$ scale divisions, but those for 1847 and 1848 were so vitiated by large disturbances (in some cases the daily differences being upwards of 100 scale divisions), that they were evidently unfitted for a good determination; and some mode of rejecting the larger disturbances was desirable. In order, however, that the same rule should be followed throughout, the differences which after correction for temperature (by $q'' = 1{\cdot}33$ scale divisions) were found to exceed 10 scale divisions, were rejected *in all cases* from November 1843 to June 1848. The following are the results:—

1844.	$q' = \frac{790{\cdot}60}{630{\cdot}43}$	$= 1{\cdot}254$	sc. div.;	prob. error	$= 0{\cdot}081$	sc. div.;	diffs. rejected	$= 2{\cdot}7$ per cent.
1845.	$q' = \frac{914{\cdot}21}{679{\cdot}83}$	$= 1{\cdot}354$	„	„	$= 0{\cdot}038$	„	„	$= 2{\cdot}0$ „
1846.	$q' = \frac{790{\cdot}83}{626{\cdot}24}$	$= 1{\cdot}263$	„	„	$= 0{\cdot}074$	„	„	$= 6{\cdot}3$ „
1847.	$q' = \frac{863{\cdot}03}{583{\cdot}99}$	$= 1{\cdot}478$	„	„	$= 0{\cdot}077$	„	„	$= 12{\cdot}0$ „
[1848].	$q' = \frac{649{\cdot}23^*}{428{\cdot}31}$	$= 1{\cdot}516$	„	„	$= 0{\cdot}054$	„	„	$= 13{\cdot}3^\dagger$ „

If we weight these results by the inverse squares of the probable errors, we obtain $q' = 1{\cdot}378$ sc. div.; but we obtain the most probable result by taking the sums of differences for the whole period, or

$$q' = \frac{4007{\cdot}90}{2948{\cdot}80} = 1{\cdot}359 \text{ sc. div.; probable error} = 0{\cdot}029 \text{ sc. div.}$$

This differs only 0·03 sc. div. from the result obtained for 1844 and 1845. There appears, however, to be an increase in the value of q' for 1847 and 1848, but whether this is a real increase (which is quite possible, if the value depends on the condition of the wires) would require a more careful discussion to determine. The daily means vary so similarly at all the stations that it only requires for such a discussion to have one station accurately corrected (or to take a station within the tropics where the temperature variations are small), when, by subtracting the corrected daily mean scale readings reduced to a common unit (rejecting days of large disturbance) from those for the station whose temperature coefficient is required, the remainders will depend almost entirely on the differences of temperature for the bifilar in question.

* This includes the six months of 1848, and the two months of 1843.

† Six months of 1848 only, no differences having been rejected in November and December 1843.

39. The following table contains the final results for each bifilar. An examination of it will show, that the true temperature coefficient is on an average only seven-tenths ($\frac{7}{10}$) of that previously obtained from experiments in water of different temperatures, and that in some cases the former is only one-half of the latter.

TABLE XVI.—VALUES OF THE BIFILAR TEMPERATURE COEFFICIENT q AS OBTAINED FROM COMPARISONS OF THE DAILY MEANS, AND FROM EXPERIMENTS IN WATER AT DIFFERENT TEMPERATURES.

Observatory.	Magnet.	Period Discussed.	Coefficient from Daily Means, $\frac{(q)}{k}=q$	Probable Error.	k.	$q'k=(q)$.	Coefficient by Water Experiments, q.	Ratio $\frac{q'k}{q}=\frac{(q)}{q}$.	Remarks.
Makerstoun,	...	1843-45	Sc. Div. 1·90	Sc. Div. ...	0·000140	0·000266	0·000304	0·875	
Cape of Good Hope.	R	Dec. 1841 – Oct. 1843	1·183	0·05	0·000180	0·000213	0·000290	0·734	q and q' for $t=63°{\cdot}6$
	R	Nov. 1843 – Oct. 1844	0·587	0·03	0·00035	0·000205	0·000287	0·714	q and q' for $t=67°$
	C_2	Nov. 1844 – June 1846	0·438	0·04	0·00022	0·000096	0·000220	0·484	...
St Helena,	...	1842-43	1·92	0·06	0·000189	0·000193	0·000260	0·742	q and q' for $t=65°$
Hobarton,	...	April 1842 – June 1843	1·520	0·05	0·000120	0·000182	0·000234	0·778	
	...	Aug. 1843 – Sept. 1848	0·692	0·02	0·000229	0·000158	0·000230	0·689	
Trevandrum,	No. 3	1842	1·56	0·15	0·000135	0·000212	0·000253	0·838	
	No. 5	1843-45	2·17	0·09	0·000144	0·000312	0·000347	0·899	
Singapore,	...	1842-45	0·70	0·04	0·000195	0·000137	0·000214	0·638	
Toronto,	...	1842	2·53	0·08	0·000074	0·000188	0·000260	0·723	
	...	Nov. 1843 – June 1848	1·36	0·03	0·000087	0·000118	0·000234	0·500	

40. *Remaining and Accidental Errors.*—I shall notice briefly, in conclusion, the other sources of error to which the instrument is liable.

1*st*, In suspending the magnet, the wires are apt to receive a twist or kinch; as the wire gradually takes the *set*, the scale readings change, indicating an increase of horizontal force if the twist has been with the torsion (v), and a diminution of force if the twist has been in the opposite direction. An example of this is to be seem in the "Makerstoun Observations," between June and September 1841.

2*d*, From the wires lying on the *side* of the screw threads instead of in the hollow. An example of this (if not of the 1*st*) is to be found in the observations made at Toronto, when the force appeared to increase $\frac{1}{18}$th of its whole value between February 24 and October 9, 1843.

3*d*, From sudden shocks, including those due to concussions of the air by thunder-claps and sudden slamming of doors. Examples of both of these were observed at the Observatory on the Agustier Mountain in Travancore.

4*th*, From ill-closed boxes, currents of air, and entry of spiders. Examples of these are unfortunately not wanting.

5*th*, From the thermometer within the box not showing the true temperature of the magnet. The error due to this cause may be great; and it is considerable in observatories where stoves have been used, and where there are necessarily sudden changes of temperature.

6*th*, From the copper ring used as a quieter, which has sometimes changed its position, and which has a variable effect in different positions of the magnet.

7*th*, From the shifting of the telescope on its pillar.

8*th*, From the approach of magnets, or articles containing iron.

All these, and other sources of error, have to be cared for, and the corrections to be applied, wherever the errors exist, if it is wished to give the observations any real scientific value.

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